Effective Potential on Fuzzy Sphere

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The effective potential of quantized scalar field on fuzzy sphere is evaluated to the two-loop level. In the one-loop approximation, we see that the Coleman-Weinberg mechanism of the radiatively symmetry breaking shown in the planar system does also appear in the sphere system. For the two-loop diagram on the fuzzy sphere, we see that there is a new finite contribution, called as noncommutative anomaly, which will survive in the commutative sphere limit. The noncommutative anomaly is found to have an inclination to break the symmetry. However, after the detail analyses, we see that the complete two-loop quantum corrections could not change the symmetry property in the tree level.

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1 Introduction

Noncommutative fuzzy sphere is known to correspond sphere D2-branes in string theory with background linear B-field [1]. In the presence of constant RR three-form potential the D0-branes are found to expand into a noncommutative fuzzy sphere configurations [2]. It also knows that the field theories on fuzzy sphere appear naturally from D-brane theory and matrix model with some backgrounds [1,3-6]

In the ordinary matrix model one could not find the fuzzy sphere solutions. However, adding a Chern-Simons term to the matrix model will enable us to describe the noncommutative fuzzy sphere as a classical solution. Comparing the energy in the various classical solutions one can find that the separated D0-branes will expand into a largest noncommutative fuzzy sphere to achieve minimum energy [2].

The problems of the one loop renormalization of the scalar field on the fuzzy sphere have been studied in some recent papers [7-10]. The phenomena of the UL/IR mixing on the fuzzy sphere are different from those in the noncommutative plane. Especially, it has found by Chu. et. al. [9] that , in the limit of the commutative sphere, the two point function is regular without UV/IR mixing; however quantization does not commute with the commutative limit, and a finite "noncommutative anomaly" survives in the commutative limit. Chu. et. al. [9] use this to provides an explanation of the UV/IR mixing as an infinite variant of the "noncommutative anomaly".

In this paper we will investigate the effective potential of quantized scalar field on fuzzy sphere to the two-loop level. In the next section, we first review the appearance of the fuzzy sphere in the matrix model with a Chern-Simons term. (Note that the Chern-Simons term may be coming from the constant RR three-form potential in the D0-branes system [2].) We then quantize the scalar field on the fuzzy sphere and set up the general formula and Feynman rule to evaluate the effective potential.

In the section 3, we begin to evaluate the one-loop potential. We first evaluate the potential on a planar system and then that on the fuzzy sphere. The result shows that the Coleman-Weinberg mechanism of the radiatively symmetry breaking could be shown in the planar system and in the fuzzy sphere system.

In the section 4, we analyze the Feynman diagrams in the two-loop potential on the fuzzy sphere. We find that there is a new finite contribution, called as noncommutative anomaly [9], which will survive in the commutative limit. However, after the detailed analyses, we show that the two-loop quantum corrections could not change the symmetry property of the tree level. The last section is devoted to a short conclusion.

2 Quantized Scalar Field and Effective Potential on Fuzzy Sphere

2.1 Fuzzy Sphere

The matrix model with Chern-Simons term [1] is described by the action:

$$S = T_0 \; Tr\Big(\frac{1}{2}\dot{X_i}^2 + \frac{1}{4}[X_i, X_j][X_i, X_j] - \frac{i}{3}\lambda_N \; \epsilon_{ijk} X_i [X_j, X_k]\Big). \tag{2.1}$$

where $X_i, i = 1, 2, 3$ are $N \times N$ matrices and $T_0 = \sqrt{2\pi}/g_s$ is the zero-brane tension. The static equations of motion are

$$[X_j, \Big([X_i, X_j] - i\lambda_N \; \epsilon_{ijk} X_k\Big)] = 0, \tag{2.2}$$

and the associated energy is

$$E = -T_0 Tr\Big(\frac{1}{4}[X_i, X_j][X_i, X_j] - \frac{i}{3}\lambda_N \epsilon_{ijk} X_i [X_j, X_k]\Big). \tag{2.3}$$

This model admits commutating solutions and static fuzzy sphere solutions [1,2]. The commutating solutions represent N D0 branes and satisfy the relations

$$[X_i, X_j] = 0, \tag{2.4}$$

which have the energy

$$E = 0. \tag{2.5}$$

The static fuzzy sphere solutions satisfy the relations

$$[X_i, X_j] = i\lambda_N \epsilon_{ijk} X_k, \tag{2.6}$$

and are described by the relations

$$X_i = \lambda_N J_i, \tag{2.7}$$

$$X_1^2 + X_2^2 + X_3^2 = R^2. \tag{2.8}$$

where J_1, J_2, J_3 define, say, the N dimensional irreducible representation of SU(2) and are labeled by the spin $\alpha = N/2$. The noncommutativity parameter λ_N is of dimension length, and can be taken positive. The radius R define in (2.8) is quantized in units of λ_N by

$$\frac{R}{\lambda_N} = \sqrt{\frac{N}{2}\left(\frac{N}{2}+1\right)}; N = 1, 2, ... \tag{2.9}$$

Besides the above solution X_i may be a direct sum of several irreducible representation of SU(2). Such a configuration could also solves the equation of motion, i.e.

$$X_i = \lambda_N \oplus_{r=1}^{s} J_i^{(r)}, \quad \sum_{r=1}^{s}(2j_r + 1) = N. \tag{2.10}$$

The energy E of these static fuzzy sphere solutions are given by

$$E = -T_0\lambda^4 \, \frac{1}{6}\sum_{r=1}^{s} J_r(J_r + 1)(2J_r + 1). \tag{2.11}$$

From the above relation it is clear that the ground state is the N-dimensional fuzzy sphere [2].

2.2 Quantized Scalar Field on Fuzzy Sphere: Feynman Rule

A field on the fuzzy sphere is defined as an algebra S_N^2 generated by Hermitian operators $\vec{X} = (X_1, X_2, X_3)$ which are described in the section 2.1. The integral of a function $F \in S_N^2$ over the fuzzy sphere is given by

$$R^2 \int F = \frac{4\pi R^2}{N+1} Tr[F(X)], \tag{2.12}$$

and the inner product can be defined by

$$(F_1, F_2) = \int F_1^\dagger F_2. \tag{2.13}$$

A scalar Φ^4 theory on the fuzzy sphere is described by the action

$$S = \int \frac{1}{2}\Phi(\Delta + m^2)\Phi + \frac{g}{4!}\Phi^4. \tag{2.14}$$

Here Φ is a Hermitian field, m^2 is the dimensionless mass square, g is a dimensionless coupling and $\Delta = \sum J_i^2$ is the Laplace operator. To evaluate the effective potential of the above model we can use the path-integration formulation of Jackiw [11].

First, we assume that there exists a stationary point at which Φ is a constant field Φ_0. Thus

$$\left.\frac{\delta S}{\delta \Phi}\right|_{\phi_0} = 0. \tag{2.15}$$

Next, we expand the Lagrangian about the stationary point and the action becomes

$$S[\Phi] = S[\Phi_0] + \frac{1}{2}\int \tilde{\Phi}(X)\ \tilde{\Phi}(Y)\ \left.\frac{\delta^2 S}{\delta\Phi(X)\delta\Phi(Y)}\right|_{\Phi_0} + \int\ \tilde{\mathcal{L}_{\mathcal{I}}}(\tilde{\Phi}, \Phi_0), \tag{2.16}$$

in which $\tilde{\Phi} \equiv \Phi - \Phi_0$ and $\tilde{\mathcal{L}_{\mathcal{I}}}(\tilde{\Phi}, \Phi_0)$ can be found from the Lagrangian Eq.(2.14).

Then, after expanding $\tilde{\Phi}$ in terms of the modes,

$$\tilde{\Phi} = \sum_{L,l} a_l^L Y_l^L, \quad L = 0, 1, ..., N; \quad -L \le l \le L, \tag{2.17}$$

in which the Fourier coefficient a_l^L are treated as the dynamical variables, the path integral quantization procedure [7] is defined by integrating over all possible configuration of a_l^L. Therefore the k-points Green's functions are computed by the relation

$$\langle a_{l_1}^{L_1} \cdots a_{l_k}^{L_k} \rangle = \frac{\int [D\tilde{\Phi}] e^{-S} a_{l_1}^{L_1} \cdots a_{l_k}^{L_k}}{\int [D\tilde{\Phi}] e^{-S}}, \tag{2.18}$$

Note that the complete basis of functions on S_N^2 is given by the $(N+1)^2$ spherical harmonics, $Y_l^L, (L = 0, 1, ..., N; -L \le l \le L)$. They correspond to the usual spherical harmonics, however the angular momentum has an upper bound N here. This is a characteristic feature of fuzzy sphere.

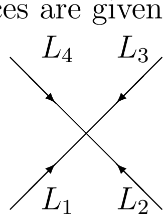
The propagator so obtained is

$$D^{-1}(\Phi_0) = \langle a_l^L a_{l'}^{L'\dagger} \rangle = (-1)^l \langle a_l^L a_{-l'}^{L'} \rangle = \delta_{LL'} \delta_{ll'} \frac{1}{L(L+1) + \mu^2}, \tag{2.19}$$

in which $a_l^{L\dagger} = (-1)^l a_{-l}^L$ and

$$\mu^2 = m^2 + \frac{1}{2} g \Phi_0^2. \tag{2.20}$$

The four-legs vertices are given by



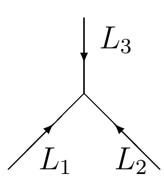
$$a_{l_1}^{L_1} \cdots a_{l_4}^{L_4} \; V_4(L_1, l_1; \cdots; L_4, l_4), \tag{2.21}$$

where

$$V_4(L_1, l_1; \cdots; L_4, l_4) = \frac{g}{4!} \frac{N+1}{4\pi} (-1)^{L_1+L_2+L_3+L_4} \prod_{i=1}^{4} (2L_i + 1)^{1/2} \sum_{L,l} (-1)^l (2L+1)$$

$$\cdot \begin{pmatrix} L_1 & L_2 & L \\ l_1 & l_2 & l \end{pmatrix} \begin{pmatrix} L_3 & L_4 & L \\ l_3 & l_4 & -l \end{pmatrix} \begin{Bmatrix} L_1 & L_2 & L \\ \alpha & \alpha & \alpha \end{Bmatrix} \begin{Bmatrix} L_3 & L_4 & L \\ \alpha & \alpha & \alpha \end{Bmatrix}. \tag{2.22}$$

The above Feynmam rule of propagator and four-legs vertices can be found in [9]. Here we need also the Feynmam rule of three-legs vertices, it is described as



$$a_{l_1}^{L_1} a_{l_2}^{L_2} a_{l_3}^{L_3} \; V_3(L_1, l_1; L_2, l_2; L_3, l_3), \tag{2.23}$$

where

$$V_3(L_1,l_1;L_2,l_2;L_3,l_3) = \frac{g}{6}\Phi_0\sqrt{\frac{N+1}{4\pi}}(-1)^{2\alpha+L_1+L_2+L_3}\sqrt{(2L_1+1)(2L_2+1)(2L_3+1)}$$

$$\begin{pmatrix} L_1 & L_2 & L_3 \\ l_1 & l_2 & -l_3 \end{pmatrix}\begin{Bmatrix} L_1 & L_2 & L_3 \\ \alpha & \alpha & \alpha \end{Bmatrix}. \tag{2.24}$$

Here the first bracket is the Wigner $3j$-symbol and the curly bracket is the $6j$–symbol of $SU(2)$, in the standard mathematical normalization [12]. Note that to derive the above Feynmam rule we have used the following "fusion" algebra [12]

$$Y_i^I Y_j^J = \sqrt{\frac{N+1}{4\pi}}\sum_{K,k}(-1)^{2\alpha+I+J+K+k}\sqrt{(2I+1)(2J+1)(2K+1)}$$

$$\begin{pmatrix} I & J & K \\ i & j & -k \end{pmatrix}\begin{Bmatrix} I & J & K \\ \alpha & \alpha & \alpha \end{Bmatrix}Y_k^K,. \tag{2.25}$$

where the sum is over $0 \le K \le N, -K \le k \le K$, and $\alpha = N/2$.

Then the effective potential $V(\phi_0)$ is found to be [11]

$$V(\Phi_0) = V_0(\Phi_0) + \frac{1}{2}\int \ln det\left[D^{-1}(\Phi_0)\right] - \ < exp\left(\int \tilde{\mathcal{L}_{\mathcal{I}}}(\tilde{\Phi},\Phi_0)\right) > . \tag{2.26}$$

The first term in Eq.(2.20) is the classical potential. The second term is the one-loop contribution from the second term in Eq.(2.16). The third term is the higher-loop contribution of the effective potential. To obtain it we shall evaluate the expectation value of the third term in Eq.(2.16) by the conventional Feynmam rule, with $D^{-1}(\Phi_0)$ as the propagator and keep only the connected single-particle irreducible graphs [11].

In next section, we use the above formula to analyze the one-loop potential, and in the section 4, we detailed evaluate the two-loop potential on the fuzzy sphere.

3 One-Loop Effective Potential

We first evaluate the one-loop effective potential of the scalar Φ^4 theory on the planar system and then do that in the sphere system.

Using the formula in [11] (or (2.26)) the one-loop effective potential on the planar system is

$$V(\Phi_0) = \frac{1}{2}\int d^2\vec{k}\ln det\left[D^{-1}(\Phi_0,\vec{k})\right] = -\frac{1}{2}\int d^2\vec{k}\ \ln\left(\vec{k}^2+\mu^2\right) = -\pi\int_0^\Lambda dkk\ \ln\left(k^2+\mu^2\right)$$

$$= -\pi\left[\left(m^2+\frac{1}{2}g\Phi_0^2\right)\ln\Lambda^2 + \left(m^2+\frac{1}{2}g\Phi_0^2\right) - \left(m^2+\frac{1}{2}g\Phi_0^2\right)\ \ln\left(m^2+\frac{1}{2}g\Phi_0^2\right)\right]. \tag{3.1}$$

Here we have introduced a momentum cutoff Λ and dropped off the irrelevant field-independent terms. From the above result we see that, in the massless model, the first term and second term are merely used to render the renormalized mass to be zero. In this case, the third term which will become negative for small Φ_0 shall break the symmetry radiatively. This is the well known Coleman-Weinberg mechanism. In the massive case, we can expand the above result around the point $\Phi_0^2 = 0$ and it is easily to see that, the one-loop result could renormalize the mass and coupling constant but does not change the symmetry in the tree level.

Now, using the formula in (2.26) the one-loop effective potential on a fuzzy sphere becomes

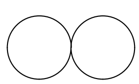
$$\frac{1}{2}\int \ln det\left[D^{-1}(\Phi_0,\vec{k})\right] = -\frac{1}{2}\sum_{L,l}\ln\left[L(L+1)+\mu^2\right] = -\frac{1}{2}\sum_{L=0}^{N}(2L+1)\ln\left[L(L+1)+\mu^2\right]$$

$$= -\frac{1}{2}\ln\left(m^2+\frac{1}{2}g\Phi_0^2\right) - \frac{1}{2}\sum_{L=1}^{N}\ln\left[L(L+1)+m^2+\frac{1}{2}g\Phi_0^2\right]. \qquad (3.2)$$

From the above result we can see the following properties. In the massless case, we expand the second term around the point $\Phi_0^2 = 0$ and it is easily to see that, they will renormalize the mass and coupling constant and give the high-power terms of Φ_0^2. However, the first term will become negative for sufficiently large Φ_0. Thus it could break the symmetry and the Coleman-Weinberg mechanism is shown. In the massive case, we can expand eq.(3.2) around the point $\Phi_0^2 = 0$ and it is easily to see that, the one-loop result could only renormalize the mass and coupling constant, and give the high-power terms of Φ_0^2, but they could not change the symmetry property in the tree level.

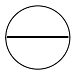
4 Two-Loop Effective Potential

In the two-loop level we have to evaluate the following two planar and two nonplanar diagrams. The contributions of the effective potential from the planar diagram are



$$= I_1^P$$

$$= -\frac{g}{4\pi}\frac{1}{3}\sum_{L}\frac{2L+1}{L(L+1)+\mu^2}\sum_{J}\frac{2J+1}{J(J+1)+\mu^2}. \qquad (4.1)$$

 $= I_2^P$

$$= -\Phi_0^2 \, (\frac{g}{4\pi})^2 \, \frac{1}{6} \, \sum_{L,J} \frac{2L+1}{L(L+1)+\mu^2} \, \frac{2J+1}{J(J+1)+\mu^2} \frac{2L+2J+1}{(L+J)(L+J+1)+\mu^2}. \quad (4.2)$$

The contributions of th effective potential from the nonplanar diagram are like those in the planar diagram, while with an extra factor. They are

$$I_1^N = \; -\frac{g}{4\pi}\frac{1}{6} \sum_{L,J=0}^{N} \frac{2L+1}{L(L+1)+\mu^2} \, \frac{2J+1}{J(J+1)+\mu^2} \, (-1)^{L+J+2\alpha} \, (2\alpha+1) \left\{ \begin{array}{ccc} \alpha & \alpha & L \\ \alpha & \alpha & J \end{array} \right\}. \quad (4.3)$$

$$I_2^N = -\Phi_0^2 \, (\frac{g}{4\pi})^2 \, \frac{1}{6} \sum_{L,J} \frac{2L+1}{L(L+1)+\mu^2} \frac{2J+1}{J(J+1)+\mu^2}$$

$$\times \, \frac{2L+2J+1}{(L+J)(L+J+1)+\mu^2} \, (-1)^{L+J+2\alpha} \, (2\alpha+1) \left\{ \begin{array}{ccc} \alpha & \alpha & L \\ \alpha & \alpha & J \end{array} \right\}. \quad (4.4)$$

To derive the above relations we have use the following identities of the $3j$ and $6j$ symbols, which can be found in [12]: The $3j$ symbols satisfy the orthogonality relation

$$\sum_{j,l} \left(\begin{array}{ccc} J & L & K \\ j & l & k \end{array} \right) \left(\begin{array}{ccc} J & L & K' \\ -j & -l & -k' \end{array} \right) = \frac{(-1)^{K-L-J}}{2K+1} \delta_{K,K'} \delta_{k,k'}, \quad (4.5)$$

assuming that (J, L, K) form a triangle. The $6j$ symbols satisfy the orthogonality relation

$$\sum_N (2N+1) \left\{ \begin{array}{ccc} A & B & N \\ C & D & P \end{array} \right\} \left\{ \begin{array}{ccc} A & B & N \\ C & D & Q \end{array} \right\} = \frac{1}{2P+1} \, \delta_{P,Q}, \quad (4.6)$$

and the following sum rule

$$\sum_N (-1)^{N+P+Q} (2N+1) \left\{ \begin{array}{ccc} A & B & N \\ C & D & P \end{array} \right\} \left\{ \begin{array}{ccc} A & B & N \\ D & C & Q \end{array} \right\} = \left\{ \begin{array}{ccc} A & C & Q \\ B & D & P \end{array} \right\}, \quad (4.7)$$

assuming that (A, D, P) and (B, C, P) form a triangle.

Using the above the formula we will in the subsection 4.1 analyze the general property of the two-loop contributions. In the subsection 4.2 we then perform the evaluations of the two-loop diagrams under the approximation that $m^2 \gg N$.

4.1 General Property of the Two-Loop Diagrams

4.1.1 Planar Diagram

The contributions of planar diagrams are easy to evaluate. The calculations are as follows.

$$I_1^P = -\frac{g}{4\pi}\frac{1}{3}\left[\sum_L \frac{2L+1}{L(L+1)+\mu^2}\right]^2 = -\frac{g}{4\pi}\frac{1}{3}\left[\frac{1}{\mu^2} + \sum_{L=1}^{N}\frac{2L+1}{L(L+1)+\mu^2}\right]^2. \tag{4.8}$$

$$I_2^P = -\Phi_0^2(\frac{g}{4\pi})^2\ \frac{1}{6}\ [\frac{1}{\mu^6} + 2\frac{1}{\mu^2}\sum_{L=1}^{N}\left[\frac{2L+1}{L(L+1)+\mu^2}\right]^2$$
$$+\sum_{L,J=1}^{N}\frac{2L+1}{L(L+1)+\mu^2}\ \frac{2J+1}{J(J+1)+\mu^2}\frac{2L+2J+1}{(L+J)(L+J+1)+\mu^2}] \tag{4.9}$$

In the massive case we can expand the above two relations around the point $\Phi_0^2 = 0$ (note that $\mu^2 = m^2 + \frac{1}{2}g\Phi_0^2$), and these terms will renormalize the mass and coupling constant and give the high-power terms of Φ_0^2. They does not change the symmetry property of the tree level. In the massless case, the terms $\propto \frac{1}{\Phi_0^n}$ and those expanding around the point $\Phi_0^2 = 0$ could at most renormalize the mass and coupling constant, and give the high-power terms of Φ_0^2, but they could not change the symmetry property of the tree level.

4.1.2 Non-Planar Diagram

For the non-planar diagrams we have the following relations

$$I_1^N = -\frac{g}{4\pi}\frac{1}{6}\ [\frac{1}{\mu^4} + 2\sum_L^N\frac{1}{\mu^2}\frac{2L+1}{L(L+1)+\mu^2}$$
$$+\sum_{L,J=1}^{N}\frac{2L+1}{L(L+1)+\mu^2}\ \frac{2J+1}{J(J+1)+\mu^2}\ (-1)^{L+J+2\alpha}\ (2\alpha+1)\left\{\begin{array}{ccc}\alpha & \alpha & L\\ \alpha & \alpha & J\end{array}\right\}]. \tag{4.10}$$

$$I_2^N = -\Phi_0^2(\frac{g}{4\pi})^2\ \frac{1}{6}[\frac{1}{\mu^6} + 2\frac{1}{\mu^2}\sum_{L=1}^{N}\left[\frac{2L+1}{L(L+1)+\mu^2}\right]^2 + \sum_{L,J=1}^{N}\frac{2L+1}{L(L+1)+\mu^2}$$
$$\times\frac{2J+1}{J(J+1)+\mu^2}\frac{2L+2J+1}{(L+J)(L+J+1)+\mu^2}(-1)^{L+J+2\alpha}\ (2\alpha+1)\left\{\begin{array}{ccc}\alpha & \alpha & L\\ \alpha & \alpha & J\end{array}\right\}]. \tag{4.11}$$

Here we have use the relation [12]

$$\left\{\begin{array}{ccc}\alpha & \alpha & L\\ \alpha & \alpha & 0\end{array}\right\} = (-1)^{2\alpha+L}\frac{1}{2\alpha+1}. \tag{4.12}$$

We thus see that, in the massless case the terms $\propto \frac{1}{\Phi_0^n}$ coming from (4.10) is just half of that from (4.8) and that coming from (4.11) is just that from (4.9). *Therefore, the differences between the planar and non-planar are only in the power of* Φ_0^2. Thus, the non-planar diagram does not change the symmetry property of the tree level. In the massive case, we can safely expand the above two relations around the point $\Phi_0^2 = 0$ (Note that $\mu^2 = m^2 + \frac{1}{2}g\Phi_0^2$), and these terms will renormalize the mass and coupling constant and give the high-power terms of Φ_0^2. They does not change the symmetry property of the tree level.

4.2 Heave Scalar Field Approximation in Two-Loop Diagram

To proceed the calculation we shall adopt some approximations, as is difficult in the summation over the index in the $6j$ symbols.

Note that to evaluate the scalar field propagator, Chu et.al. [9] consider the case that the external moment $L \ll \alpha$. In this case they used the relation [12]

$$\left\{ \begin{array}{ccc} \alpha & \alpha & L \\ \alpha & \alpha & J \end{array} \right\} \approx (-1)^{\frac{L+J+2\alpha}{2\alpha}} P_L \left(1 - \frac{J^2}{2\alpha^2}\right), \qquad L \ll \alpha, \tag{4.13}$$

to evaluate the one-loop correction to the mass square. They have found that the noncommutative anomaly, which will survive in the commutative sphere limit, is $-\frac{g}{12\pi}\sum_{k=1}^{L}\frac{1}{k}$.

As we will perform the two-loop calculation in which the summations in (4.3) and (4.4) are over all the values of L, $0 \le L \le N$, the approximation of (4.13) can not be used now. To proceed, we will perform the evaluations of the two-loop diagrams under the approximation that $m^2 \gg N$.

4.2.1 Planar Diagram

It is easy to perform the approximation in the planar diagrams. The results are

$$\begin{aligned} I_1^P &= -\frac{g}{4\pi}\frac{1}{3} \sum_{L,J=0}^{N} \frac{2L+1}{\mu^2}\left(1 - \frac{L(L+1)}{\mu^2} + \cdots\right)\frac{2J+1}{\mu^2}\left(1 - \frac{J(J+1)}{\mu^2} + \cdots\right) \\ &= -\frac{g}{4\pi}\frac{1}{3}\left[\frac{(N+1)^4}{\mu^4} - \frac{N(N+2)(N+1)^4}{\mu^6} + O\left(\frac{1}{\mu^8}\right)\right]. \end{aligned} \tag{4.14}$$

$$I_2^P = -\Phi_0^2\,(\frac{g}{4\pi})^2\,\frac{1}{6}\left[\frac{1}{\mu^6}\left(\frac{2}{3}N(N+1)^3(N+5) + 4N\right) + O\left(\frac{1}{\mu^8}\right)\right]. \tag{4.15}$$

4.2.2 Non-Planar Diagram

The contributions of non-planar diagrams are calculated as follows.

$$\begin{aligned} I_1^N = & -\frac{g}{4\pi}\frac{1}{6}\sum_{L,J=0}^{N}\frac{2L+1}{\mu^2}\left(1-\frac{L(L+1)}{\mu^2}+\cdots\right)\frac{2J+1}{\mu^2}\left(1-\frac{J(J+1)}{\mu^2}+\cdots\right) \\ & \times(-1)^{L+J+2\alpha}\,(2\alpha+1)\left\{\begin{array}{ccc}\alpha & \alpha & L\\ \alpha & \alpha & J\end{array}\right\} \\ = & -\frac{g}{4\pi}\frac{1}{6}\left[\frac{(N+1)^2}{\mu^4}+\frac{0}{\mu^6}+O\left(\frac{1}{\mu^8}\right)\right]. \end{aligned} \tag{4.16}$$

To obtain the above results we have used the formula [12]

$$\sum_L(2L+1)\,(-1)^{L+2\alpha}\left\{\begin{array}{ccc}\alpha & \alpha & L\\ \alpha & \alpha & J\end{array}\right\}=\delta_{J,0}(2\alpha+1). \tag{4.17}$$

Now using the above formula and a simple relation $(2L+1)(2J+1)(2L+2J+1)=4(2L+1)J(J+1)+(2L+1)+2L(2L+1)(2J+1)$ we have another result

$$\begin{aligned} I_2^N = & -\Phi_0^2\left(\frac{g}{4\pi}\right)^2\frac{1}{6}\sum_{L,J}\frac{2L+1}{\mu^2}\left(1-\frac{L(L+1)}{\mu^2}+\cdots\right)\frac{2J+1}{\mu^2}\left(1-\frac{J(J+1)}{\mu^2}+\cdots\right) \\ \times & \frac{2L+2J+1}{\mu^2}\left(1-\frac{(L+J)(L+J+1)}{\mu^2}+\cdots\right)(-1)^{L+J+2\alpha}\,(2\alpha+1)\left\{\begin{array}{ccc}\alpha & \alpha & L\\ \alpha & \alpha & J\end{array}\right\} \\ = & -\Phi_0^2\,(\frac{g}{4\pi})^2\,\frac{1}{6}\left[\frac{(N+1)^2}{\mu^6}+O(\frac{1}{\mu^8})\right]. \end{aligned} \tag{4.18}$$

From the above results we see that the noncommutative anomaly contribution is

$$\begin{aligned} I_1^N+I_2^N-\frac{1}{2}I_1^P-I_2^P &= -\frac{g}{4\pi}\frac{1}{6}\left[\frac{(N+1)^2-(N+1)^4}{\mu^4}+O\left(\frac{1}{\mu^6}\right)\right] \\ &= \frac{g}{4\pi}\frac{1}{6m^4}\left[(N+1)^4-(N+1)^2\right]\left(1-g\Phi_0^2+\cdots\right)+O(\frac{1}{m^6}). \end{aligned} \tag{4.19}$$

Therefore *the noncommutative-anomaly contribution to the mass square renormalization is negative and has an inclination to break the symmetry.* However, we also see that

$$\begin{aligned} I_1^N+I_2^N+I_1^P+I_2^P &= -\frac{g}{4\pi}\frac{1}{6}\left[\frac{(N+1)^2+2(N+1)^4}{\mu^4}+O\left(\frac{1}{\mu^6}\right)\right] \\ &= \frac{g}{4\pi}\frac{1}{6m^4}\left[2(N+1)^4+(N+1)^2\right]\left(-1+g\Phi_0^2+\cdots\right)+O\left(\frac{1}{m^6}\right). \end{aligned} \tag{4.20}$$

Thus, *the complete two-loop effects are to give a positive correction to the mass square term and does not have an inclination to break the symmetry in the tree level.*

5 Conclusion

It is known that the UV/IR mixing is an intriguing property for a quantized field on the noncommutative plane [13]. As the mixing property may threaten the renormalizability and even the existence of a quantized field, many papers had studied the property [14]. On the other hand, there is no UV/IR mixing for a QFT on the fuzzy sphere [9], as a compact manifold provides a natural infrared cutoff. However, the noncommutative anomaly, which will survive in the commutative sphere limit, will be shown [9]. Thus, it is useful to study the problems of how the noncommutative anomaly affect the physical property. In this motivation, we study the effects of noncommutative anomaly on the symmetry property.

We study the effective potential of quantized scalar field on fuzzy sphere to the two-loop level. We have seen that the noncommutative character does not enter into the one-loop diagram and the Coleman-Weinberg mechanism of the radiatively symmetry breaking could be also shown in the fuzzy sphere system. In the two loop diagram, we analyze the two type Feynman diagrams therein. We see that the noncommutative anomaly [9], which will survive in the commutative limit has an inclination to break the symmetry. However, after the detailed analyses, we see that the two-loop quantum corrections could not change the symmetry property of the tree level.

Many interesting investigations are remained to be studied. These include the problems of how the noncommutative anomaly will affect the particle scattering? How the noncommutative anomaly will affect the anomalous magnetic moment of Dirac particle and particle production from a external EM field? $\cdots$. We will study these problems in the near future.

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